A Quick Look At Low Mach Number Methodology

Ann Almgren

Center for Computational Sciences and Engineering
Lawrence Berkeley National Laboratory
asalmgren@lbl.gov

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Low Mach Number: Why?

For explosions themselves, it is critical to resolve shocks and supersonic phenomena. However, during the period preceding an explosion, the flow may be low Mach number, and a low Mach number method can be useful.

- Simulations of Type Ia supernovae are very sensitive to the location and number of ignition points ("hot spots")
- To understand the conditions in the star at the time of ignition, we need to simulate hours instead of seconds
- Dynamics of convection are driven by perturbational density which is much smaller than the background density

Low Mach Number formulation

A low Mach number method:

- exploits natural separation of scales between fluid motion (at speed $U \ll c$) and acoustic wave propagation (at speed c)
- guarantees hydrostatic balance of the background state (by construction).
- takes a much larger time step and still captures the relevant physics.

Recall that all currently-available hydro codes are parallel in space but serial in time – low Mach number approximation costs more per time step but takes many fewer time steps.

Low Mach Number Asymptotics

Asymptotic analysis shows that:

$$p(\mathbf{x}, t) = p_0(r, t) + \pi(\vec{x}, t) \quad \text{where} \quad \pi/p_0 \sim \mathcal{O}(M^2)$$

- p_0 affects only the thermodynamics; π affects only the local dynamics,
- Physically: acoustic equilibration is instantaneous; sound waves are "filtered" out
- Mathematically: resulting equation set is no longer strictly hyperbolic; a constraint equation is added to the evolution equations
- Computationally: time step is dictated by fluid velocity, not sound speed.

Anelastic methods are one type of low Mach number method. They can include

- Background stratification
- Nonideal equation of state

but require that perturbations in density and temperature be small.

For modeling ignition in a white dwarf, we need a more general model that also allows:

- Finite perturbations in density and temperature
- Local compressibility from heat release and compositional changes
- Overall expansion of the star

Low Mach Number Equations

Our basic hydrodynamic equations now look like

Mass	$\rho_t + \nabla \cdot \rho U = 0$
Momentum	$(\rho U)_t + \nabla \cdot (\rho U U + \pi) = (\rho - \rho_0)\vec{g}$
Energy	$(\rho h)_t + \nabla \cdot (\rho U h) = \nabla \cdot \kappa \nabla T$
Species	$(\rho X_m)_t + \nabla \cdot (\rho U X_m) = \dot{\omega}_m$

This new system still needs to be "closed"

- EOS is transformed into a constraint on the velocity field:

$$\nabla \cdot (\beta_0 U) = \beta_0 S$$

- Reactions – same as always, though we are taking a bigger time step so coupling can become an issue
- Gravity – here it is well defined as function of $\rho_0(r, t)$

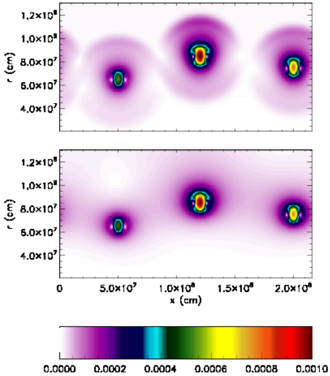
Rising Bubbles

Mach number of rising bubbles

as calculated with a compressible (above)

vs low Mach number (below) algorithm.

- Features of the bubbles themselves are identical
- $M \approx 10^{-4}$ so pressure difference is $O(10^{-8})$



For more information, see

http://www.astro.sunysb.edu/mzingale/Maestro/

https://ccse.lbl.gov/Research/MAESTRO